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SOUND PROPAGATION OVER UNEVEN GROUND AND IRREGULAR TOPOGRAPHY

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by

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INTRODUCTION

The goal of this research is to develop theoretical, computational, and experimental techniques for predicting the effects of irregular topography on long range sound propagation in the atmosphere. Irregular topography here is understood to imply a ground surface that (1) is not idealizable as being perfectly flat or (2) that is not idealizable as having a constant specific acoustic impedance. The interest of this study focuses on circumstances where the propagation is similar to what might be expected for noise from low-altitude air vehicles flying over suburban or rural terrain, such that rays from the source arrive at angles close to grazing incidence.

The objectives of the project, the experimental facility, and the early progress up through January 1988 have been described in the six previous semiannual reports [1-6]. The present report discusses those activities and developments that have resulted during the period, January 1988 through August 1988.

PERSONNEL

In addition to A. D. Pierce, Yves H. Berthelot, (the two co-principal investigators on the project), a visiting scholar, Professor Ji-xun Zhou of the Acoustics Institute of the Academy of Sciences of China (Beijing), has been working on the project since May 1987. Last fall, James A. Kearns, the graduate student working on the project under the supervision of Dr. Berthelot, successfully defended his thesis proposal (a requirement of the Graduate School of Georgia Tech for the Ph. D. degree); he is expected to finish his doctoral dissertation in 1989. During the past reporting period, Dr. Berthelot, Professor Zhou, and James Kearns have been mainly concerned with the experimental and computational phases of the project, while Dr. Pierce has been working primarily on the theoretical aspects.

COMPUTATIONAL STUDIES OF THE DIFFRACTION INTEGRAL

The first quarter (February-April) of 1988 was devoted mainly to refine the computational aspects of the research project, while the second quarter (May-July) was essentially spent on collecting data for the two hills diffraction problem. (See the following section). As indicated in the previous report [6], the theory of Matched Asymptotic Expansions (MAE) for predicting sound diffration by a ridge of finite impedance is a powerful tool which is now well understood. In many cases, the so-called Fock-Van der Pol-Bremmer (FvdPB) diffraction integral [6] reduces to some very simple expressions which are easily evaluated numerically on a computer. Such is the case, for instance, in the penumbra region, or far behind the ridge, or far above the ridge, or even in the deep acoustic shadow. However, it is sometimes necessary to evaluate the FvdPB diffraction integral in its complete form, without using any simplifying approximations. Not only is it necessary to do so in the transition regions (e.g., between the penumbra and the deep shadow), but it is also an excellent test of the validity of the computer codes previously written to compute diffraction effects in the limiting cases mentionned above.

The global structure of the general computer program used to evaluate diffraction effects by a single ridge of finite impedance is shown in Fig. 1. The input data is used to determine the region in which the receiver is located: outer region, penumbra, deep shadow, or somewhere in between these limiting cases. Whenever possible, the simplified expressions for the diffraction integral are used. For instance, in the outer region, one can use geometrical ray acoustics. In the penumbra region, it is computationally very efficient to use the knife-edge diffraction formula corrected or not by the so-called background integrals, depending on whether the receiver is far enough from the ridge. In the deep shadow zone, the creeping wave series converges rapidly so that only a few terms are needed to evaluate the diffraction integral. If the receiver is located at a point where it is inappropriate to make these simplifying assumptions, then the complete FvdPB diffraction integral is evaluated numerically. The result of these computational studies are shown in Fig. 2. The plot in Fig. 2 represents the insertion loss of the ridge as a function of receiver height when the receiver is located 64 cm behind the apex of the ridge (radius of curvature $R = 2.5$ m), for a frequency $f = 10$ kHZ. The height is normalized in a manner similar to that described in the previous report [6]. The computational result presented in Fig. 2 are obtained for the case where the receiver is located 64 cm behind the apex of the ridge. The normalized impedance of the surface (plywood covered with carpet) is measured to be $q = 1.34e^{i\pi/4}$, which scales adequately a grass-covered hill at 1 kHz. It can be seen from Fig. 2 that the limiting cases are in fact properly matched by the complete FvdPB integral.

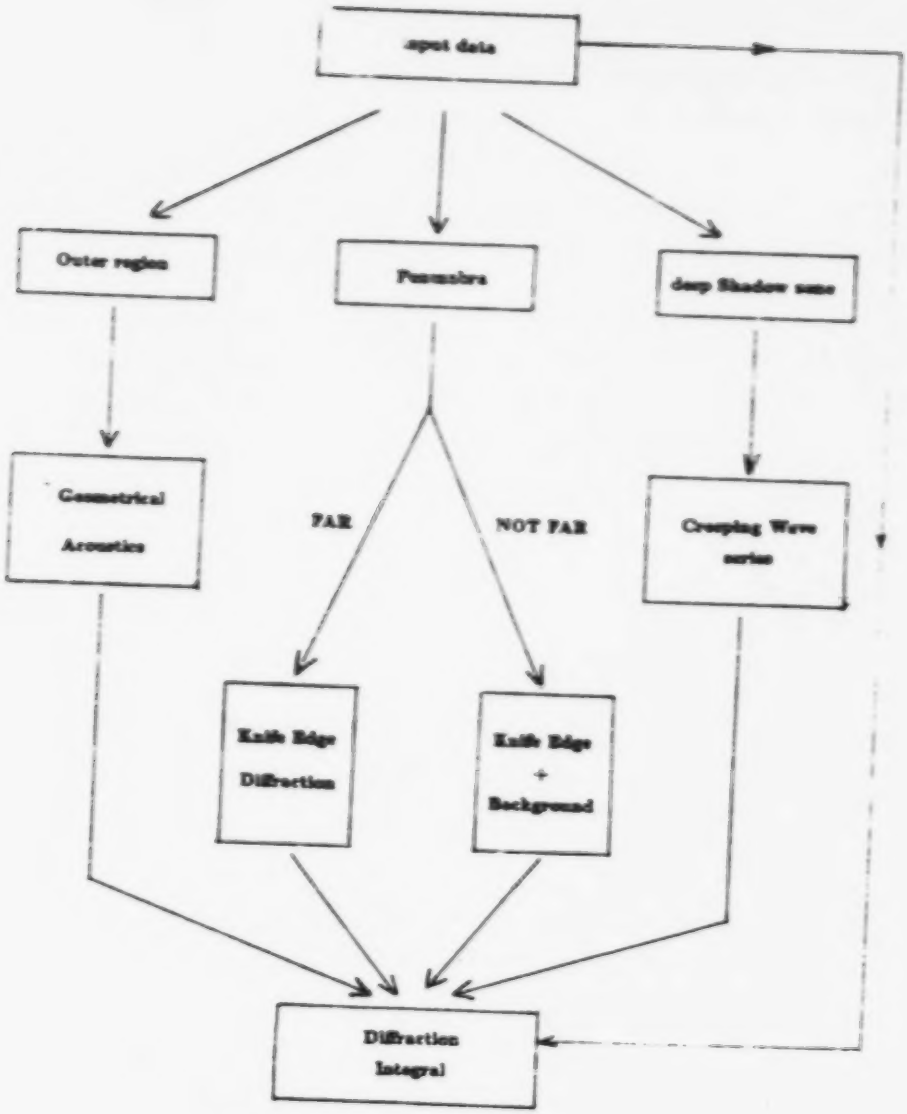

Figure 1. Block-diagram of the strategy used to compute the diffraction integral.

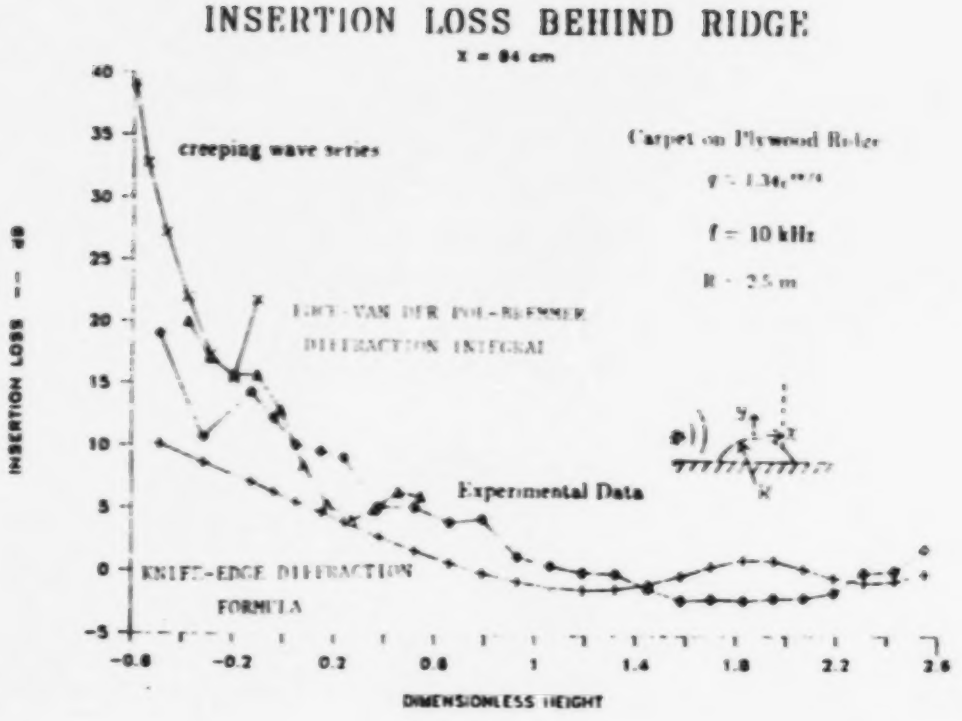

Figure 2. Insertion loss of a single ridge of finite impedance (carpet) as a function of height, measured 64 cm behind the apex. The results indicate that the different schemes used to evaluate the diffraction integral in different regimes blend well and are in good agreement with experimental data.

DIFFRACTION OF SOUND BY TWO HILLS

A new set of scale model experiments, involving a series of two ridges, were conducted using the same technique and approach as that used in the single ridge experiments (see previous status reports [1-6]). The same spark source (acoustic) and data collection system were used as before. However, a second cylindrical plywood ridge was placed directly behind and parallel to the original ridge. The two ridges are of equivalent dimensions (see Fig. 3). Measurements of the pulsed arrivals were made 1) along the surface of the second ridge, and 2) along a vertical line 120 cm downstream from the apex of the second ridge. As before, these measurements were made first with bare plywood ridge surfaces, and second with a thin carpet overlay. Samples of the reduced data are shown in Figures 4-7. In these figures, the data is represented in terms of insertion loss plotted against the same dimensionless distances obtained from the theory for a single ridge [6]. Some of the scatter in these plots is believed to be due to a low signal-to-noise ratio. This belief is particularly reasonable for the carpeted ridge data since the signal attenuation is high in these cases.

Currently, no theory is available with which to compare the experimental data. However, based upon the results of the single ridge study, certain qualitative results are expected for the two ridge experiments. For example, signal levels in the deep shadow of each ridge are expected to decay exponentially with shadow depth. Therefore, one would also expect the insertion loss measured along the surface of the second ridge to be large for large $|\xi|$. Figures 6-7 tend to confirm this expectation. Further, one might expect to observe over the second ridge pulsed arrival(s) which were diffracted by the first ridge and reflected from the forward surface of the second ridge. Figure 8 shows what is believed to be just such an arrival. We are currently in the process of reducing and analyzing the data obtained in the two-hill diffraction experiment.

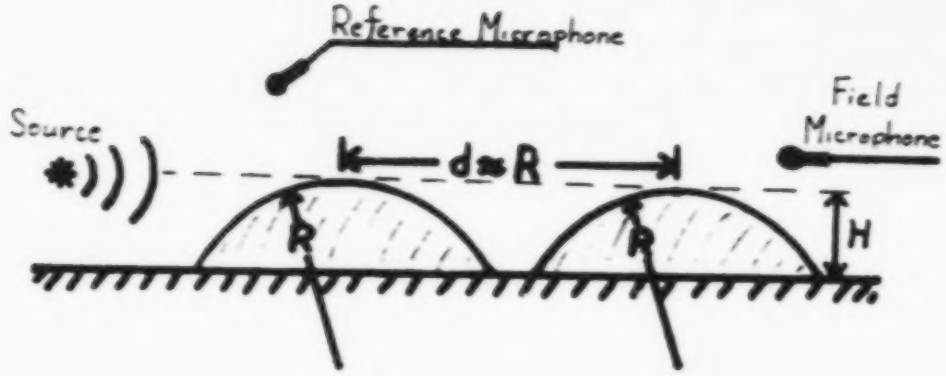

Figure 3. Schematic of the two-ridge experimental configuration. The radius of curvature and height of each ridge are R = 2.5 m, and H = 32 cm, respectively.

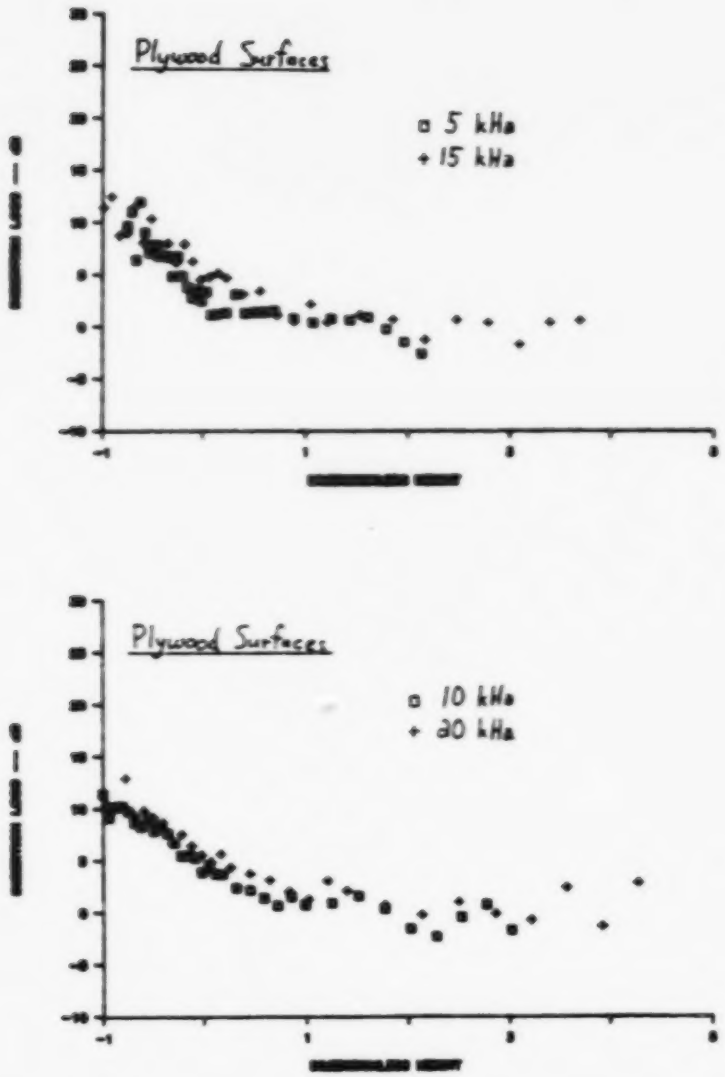

Figure 4. Insertion loss versus the dimensionless height, χ. The data was taken along a vertical line located 120 cm downstream from the apex of the second ridge. This data is for bare plywood surfaces. The top plot shows data at 5 kHz (□) and 15 kHz (+). The bottom plot shows the data at 10 kHz (□) and 20 kHz (+).

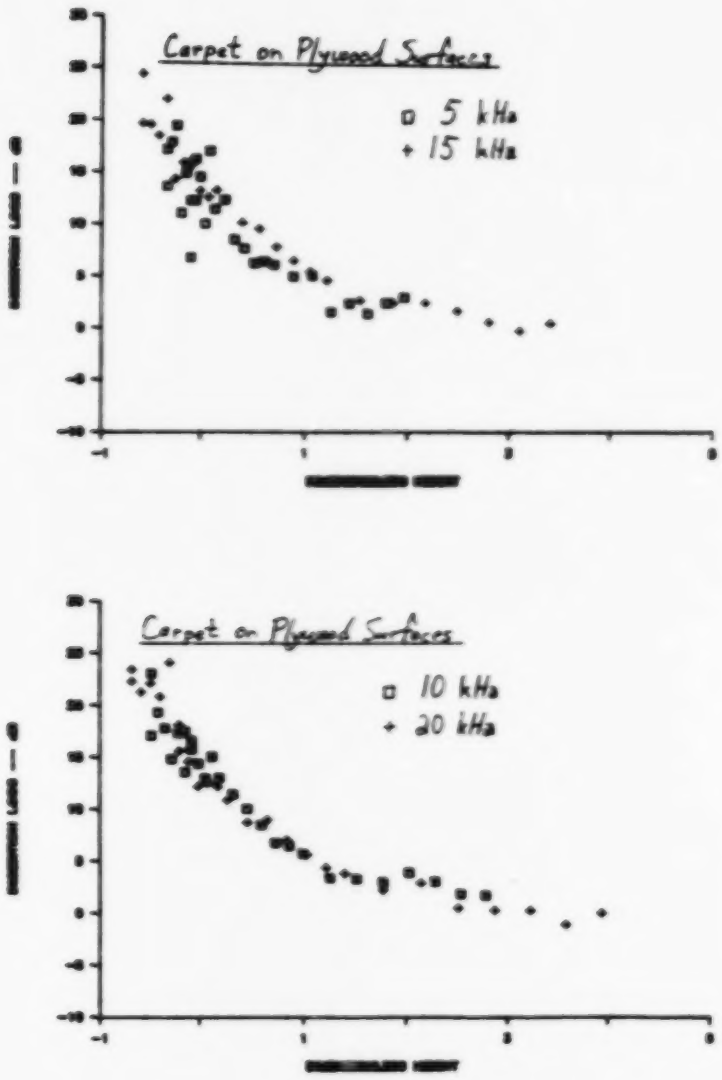

Figure 5. Insertion loss versus the dimensionless height, χ. The data was taken along a vertical line located 120 cm downstream from the apex of the second ridge. This data is for carpeted surfaces. The top plot shows data at 5 kHz (□) and 15 kHz (+). The bottom plot shows the data at 10 kHz (□) and 20 kHz (+).

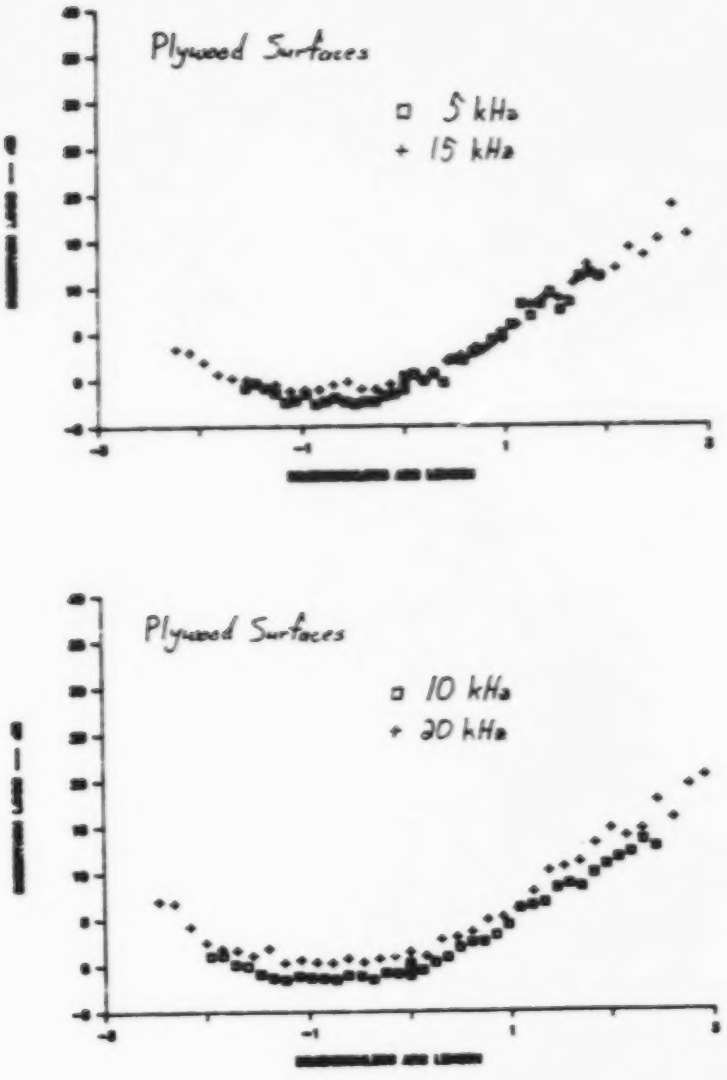

Figure 6. Insertion loss versus the dimensionless arclength, ξ. This data is for bare plywood surfaces. The top plot shows data at 5 kHz (□) and 15 kHz (+). The bottom plot shows the data at 10 kHz (□) and 20 kHz (+).

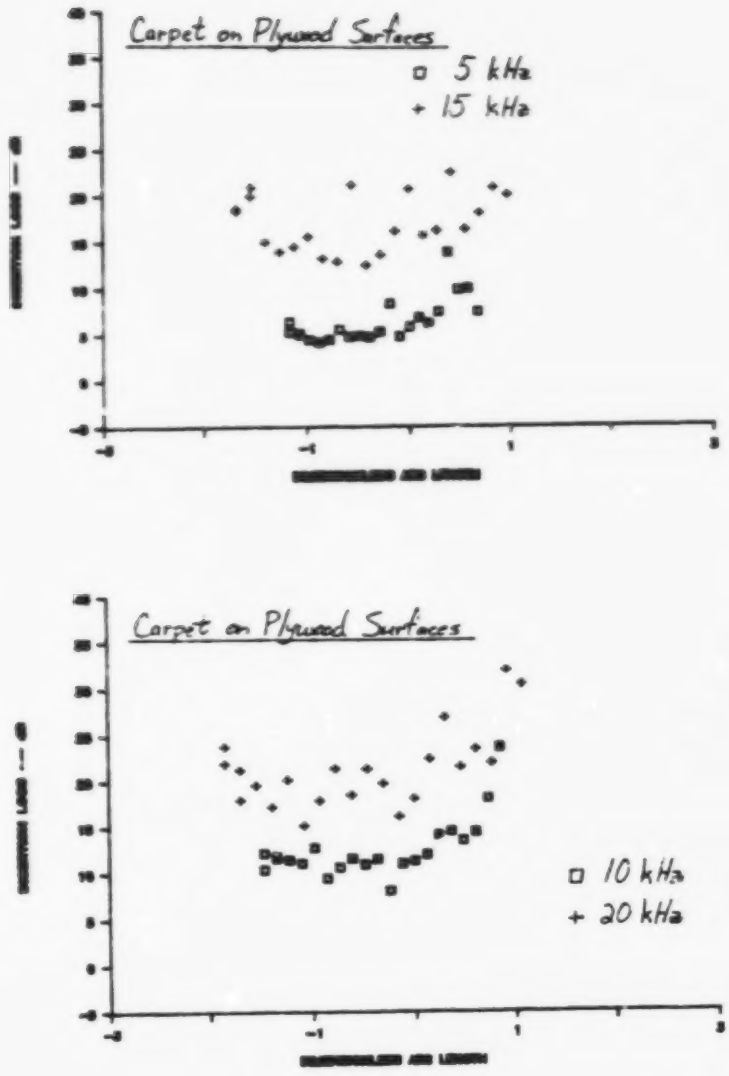

Figure 7. Insertion loss versus the dimensionless arclength, ξ. This data is for carpeted surfaces. The top plot shows data at 5 kHz (□) and 15 kHz (+). The bottom plot shows the data at 10 kHz (□) and 20 kHz (+).

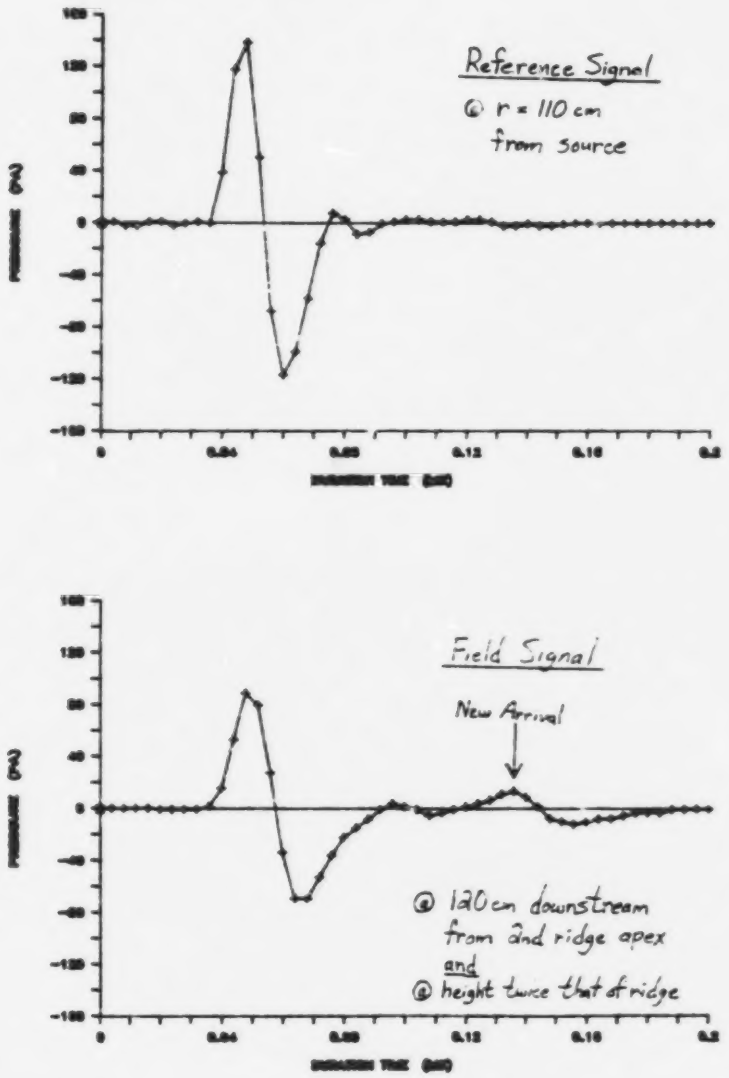

Figure 8. Arrival pulses at the reference microphone (top) and at the field microphone (bottom). The reference microphone is located 110 cm from the source and such that no reflections are captured. In this figure, the field microphone is located 120 cm downstream from the second ridge and at a height twice that of the ridge. Correction for spherical spreading losses has been made. Notice the trailing pulse arrival in the bottom figure. Due to its position and magnitude, this arrival is believed to have been diffracted by the first ridge and then reflected by the forward surface of the second ridge.

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PAPER GIVEN AT THE 3rd INTERNATIONAL SYMPOSIUM ON LONG RANGE SOUND PROPAGATION

The following pages reprint a paper written by Yves H. Berthelot, Allan D. Pierce, Ji-Xun Zhou, and James A. Kearns during reporting period. The proper citation for this paper is as follows:

Yves H. Berthelot, Allan D. Pierce, Ji-Xun Zhou, and James A. Kearns. "Diffraction effects in the long range propagation of sound in the presence of a ridge." *Proceedings of the 3rd International Symposium on Long-Range Sound Propagation and Coupling Into the Ground*, Jackson, MS, February 1988. (To be published in 1988).

Diffraction Effects in the Long Range Propagation of Sound in the Presence of a Ridge

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Abstract: It is shown that the diffraction of sound over a hill is similar to the diffraction by a sharp edge, provided that the receiver is far enough behind the hill. The shape and the impedance of the hill have virtually no influence on the acoustic field downhill. Close to the ridge, a more accurate diffraction formula can be derived from the theory of matched asymptotic expansions (MAE theory) and good agreement with experimental data is obtained. It is also shown that, in the penumbra region, the MAE theory matches the data of Berry and Daigle [submitted to *J. Acoust. Soc. Am.* (1987)] better than computations based on a creeping wave series.

Introduction

It is often desirable to predict the sound field radiated by outdoor sources of noise. An example which has already received some attention [1,2] involves the propagation over several kilometers of the low frequency noise radiated by a wind turbine. Another example of current interest deals with the noise radiated by low-altitude flying aircrafts. An important aspect of this class of problems is the diffraction associated with sound waves traveling over hills, ridges, and valleys. In an attempt to gain a better understanding of the diffraction effects involved, laboratory scaled model experiments are being conducted to study the sound field over a single smooth ridge of finite acoustic impedance. The theoretical framework for the study [3] is that of the Matched Asymptotic Expansions (MAE theory) combined with the diffraction theory of Fock [4] for electromagnetic waves. The important results from the theory are summarized below in the first section. A brief description of the scaled model experimental technique is given in the second section together with a comparison between theoretical predictions and experimental data for circumstances where the receiver is behind the hill in the acoustic penumbra, (i.e., in the transition region between the bright zone and the deep shadow zone, as indicated in Fig. 1.) The main conclusions of the study are summarized in the last section.

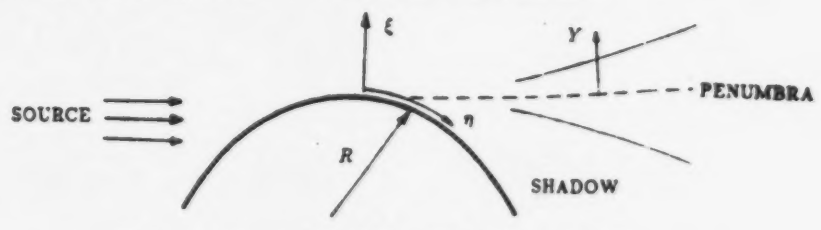

Figure 1: Geometry of the diffraction problem

Theory

The general solution of the diffraction problem corresponding to the case of a plane wave incident at grazing angle over a curved surface of radius of curvature R and of acoustic impedance Z_s is such that the Fourier transform of the acoustic pressure is directly proportional to a dimensionless function $G(\xi,\eta,q)$ referred to as the diffraction integral. The spatial coordinates ξ and η are the parabolic cylinder coordinates (see Fig. 1) nondimensionalized in terms of both the radius of curvature of the ridge R and the wavenumber k of the sound source.[3] The quantity $q = i(kR/2)^{1/3}\rho c/Z_s$ is a dimensionless number characteristic of the reciprocal of the acoustic impedance of the diffracting surface Z_s. (ρc is the characteristic impedance of the fluid medium above the ridge.) The diffraction integral is given by [3]:

$$G(\xi,\eta,q) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty} e^{i\alpha\xi}\left[v(\alpha-\eta) - \frac{v'(\alpha)-qv(\alpha)}{w_1'(\alpha)-qw_1(\alpha)}w_1(\alpha-\eta)\right]d\alpha, \qquad (1)$$

where $v(\zeta)$ and $w_1(\zeta)$ [and their derivatives, denoted here by a prime] are the Fock functions defined in terms of the Airy function of complex argument by:

$$v(\zeta) = \pi^{1/2}\mathrm{Ai}(\zeta) \qquad \text{and} \qquad w_1(\zeta) = 2\pi^{1/2}e^{i\pi/6}\mathrm{Ai}(\zeta e^{i2\pi/3}). \qquad (2)$$

Equation (1) can be simplified in some limiting cases. In the deep shadow, for instance, Eq. (1) reduces to the creeping wave series given in Ref. [5]. But for practical applications, the most interesting case is that of a receiver located in the penumbra region, where Eq. (1) becomes:

$$\begin{aligned} G = {} & e^{-i\xi^3/3}e^{i\xi\eta} - e^{i(2/3)\eta^{3/2}}\left[H(Y)e^{-i(\pi/2)Y^3} - \frac{1+i}{2}A_D(Y)\right] \\ & - \frac{1}{\sqrt{\pi}}\int_0^\infty e^{i\alpha\xi}e^{i2\pi/3}\frac{v'(\alpha)-qe^{i2\pi/3}v(\alpha)}{w_2'(\alpha)-qe^{i2\pi/3}w_2(\alpha)}w_2(\alpha-\eta e^{-i2\pi/3})d\alpha \\ & - \frac{1}{\sqrt{\pi}}\int_0^\infty e^{i\alpha\xi}\frac{v'(\alpha)-qv(\alpha)}{w_1'(\alpha)-qw_1(\alpha)}w_1(\alpha-\eta)d\alpha, \end{aligned} \qquad (3)$$

where $Y = -(2/\pi)^{1/2}\eta^{1/4}(\xi-\eta^{1/2})$ is a dimensionless height ($Y<0$ in the shadow zone), which is reasonably small; $H(\zeta)$ is the Heaviside step function; and $A_D(Y)$ is an integral occuring in many diffraction problems.[5] In equation (3), the Fock function $w_2(\zeta)$ is defined by

$$w_2(\zeta) = 2\pi^{1/2}e^{-i\pi/6}\mathrm{Ai}(\zeta e^{-i2\pi/3}). \qquad (4)$$

The first term in Eq. (3) comes from the asymptotic matching of the outer solution (far above the ridge) to the incident plane wave ($e^{i\xi\eta}$); the term in brackets is characteristic of the diffraction by sharp edges [5]; and the two remaining integrals in Eq. (3) depend on the impedance of the diffracting surface. As shown in the following section, these two terms do not contribute significantly to the diffracted field far behind the ridge.

Experimental Results

Laboratory scale experiments are being conducted with an impulsive sound source (electric spark) of effective frequency range 3-30 kHz. The diffracting surface is an arc of cylinder with a radius of curvature of about 2.5 m made of plywood covered with carpet. The impedance of the carpet is determined from the single parameter model of Delany and Bazley [6] following a technique described by Embleton *et al.* [7] Further improvements may be achieved by using the four parameters model of Attenborough.[8] After proper frequency scaling, it is found that the diffracting surface adequately models a grass covered hill. The insertion loss of the hill is obtained [9,10] from the ratio of the Fourier transforms of the acoustic pressures recorded in effect with and without the hill. Figure 2a shows the insertion loss of the ridge expressed in decibels referenced to the free field (no hill) versus the nondimensional height Y. The circles represent data points, recorded at a frequency of 20 kHz, at an horizontal distance of 2 m behind the apex of the ridge. The solid line is obtained directly from the theory of knife-edge diffraction [5] (which is basically represented by the bracketed term in Eq. (3).) Figure 2b shows a similar plot for circumstances where the receiver is closer to the apex, at an horizontal distance of 0.64 m. As expected, the chain-dashed line (knife-edge diffraction) under-estimates the recorded data but the solid line (Eq. (3)) fits quite well the data.

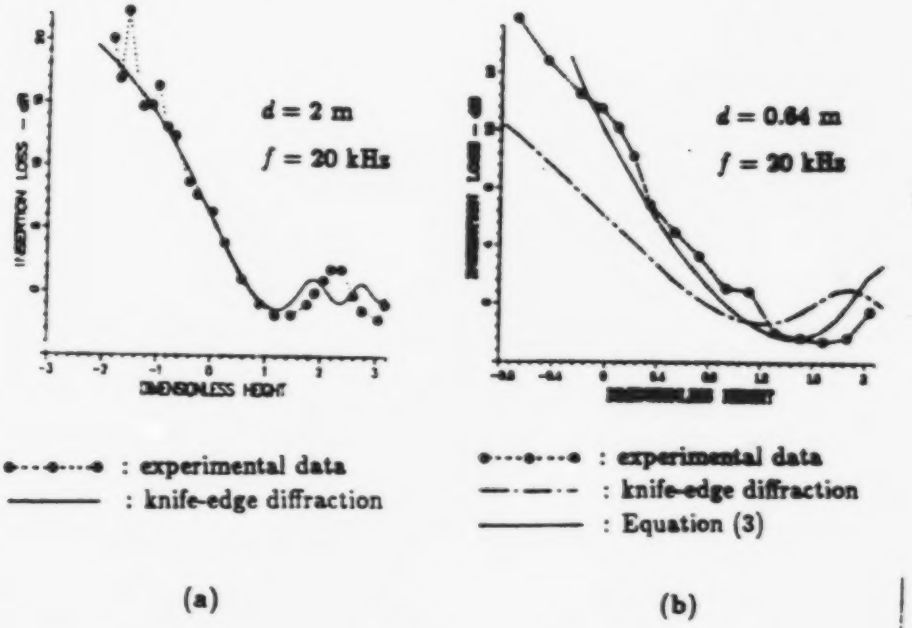

Fig. 2: Insertion loss as a function of dimensionless height Y
(a) Far downhill [d = 2m]; (b) Close to the apex [d = 0.64 m]

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In the deep shadow zone, Eq. (1) reduces to the creeping wave series discussed in Ref. [5]. Berry and Daigle [11] have recently proposed a more general form of the creeping wave series which should be valid in the penumbra region. Although theory and data agree to typically 0.5 dB in the deep shadow zone, they report a discrepancy of about 2 to 5 dB in the penumbra region. Figure 3 shows that the MAE theory is more appropriate than calculations based on the creeping wave series whenever the receiver is in the penumbra region. Figure (3) is taken directly from [11] with the addition of a dotted line representing the relative sound pressure level predicted from the MAE theory [Eq. (3) of the present paper.] The dots in Fig. (3) represent experimental data points; the solid line is the prediction from the creeping wave series solution.[11] The geometry associated with Fig. (3) is such that the source and the receiver are placed symmetrically in a straight line grazing the apex of the ridge. MAE theory and experimental data are shown to be within 0.5 dB for frequencies above 1 kHz. The 2 dB discrepancy which occurs at low frequencies is attributed to the fact that the impedance of the diffracting surface is assumed to follow the one parameter model of Delany and Bazley [6] with an effective flow resistivity of $\sigma = 60$ cgs-Rayls. As shown in Ref. [11], better agreement over the whole frequency range can be achieved by using the four parameter model of Attenborough.[8] It should be noted that there are no free parameters in the MAE theory (dotted line in Fig. 3); agreement between theory and data is better than anticipated considering the fact that the MAE theory in its present form assumes that the incident wave is a planar and not spherical as in the case of a point source. In light of Fig. (3), one can conclude that, for situations where the receiver is in the penumbra region, the MAE theory is more appropriate than calculations based on a creeping wave series.
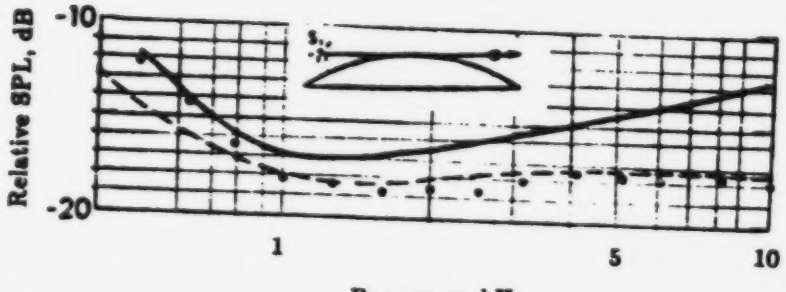
Figure 3 : Relative Sound Pressure Level in the penumbra

• • • : Data from Berry and Daigle [11]

——— : Prediction from the creeping wave series [11]

- - - - - : Prediction from MAE theory [Eq. (3)]

Conclusions

The sound field diffracted by a hill is similar to the sound field diffracted by a sharp edge, provided that the receiver is far enough downhill. In such a case, the insertion loss is independent of the shape and of the impedance of the hill. When the receiver is close to the apex of the ridge, a more complicated diffraction formula [Eq. (3)] fits very well the data. Also, the 2 to 5 dB discrepancy between theory and data reported in Ref. [11] for situations where the receiver is in the penumbra region, in the line of sight of the source, is satisfactorily explained when using the MAE theory instead of a creeping wave series. It is conjectured that the creeping wave series converges only in the region of deep shadow and that, in the penumbra region, the theory of matched asymptotic expansions is more appropriate for predicting the diffracted sound field.

Acknowledgements

The authors thank Gilles Daigle, John Preisser and William Willshire, Jr. for helpful discussions during the course of this research. The work reported here was supported by NASA, Langley Research Center.

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VIEWGRAPHS USED IN PRESENTATION OF THE PAPER PRESENTED AT THE 3rd INTERNATIONAL MEETING ON LONG RANGE SOUND PROPAGATION

The following pages reproduce the viewgraphs that were used by Yves Berthelot in his presentation at the 3rd international symposium on long range sound propagation, Jackson, MS, February 1988, entitled: "Diffraction effects in the long range propagation of sound in the presence of a ridge."

The text of this paper appears in the preceding section.

DIFFRACTION EFFECTS IN THE LONG RANGE PROPAGATION OF SOUND IN THE PRESENCE OF A RIDGE

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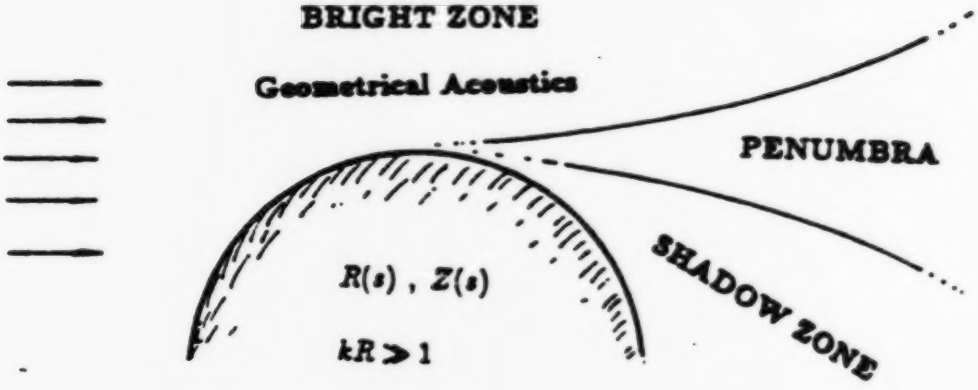
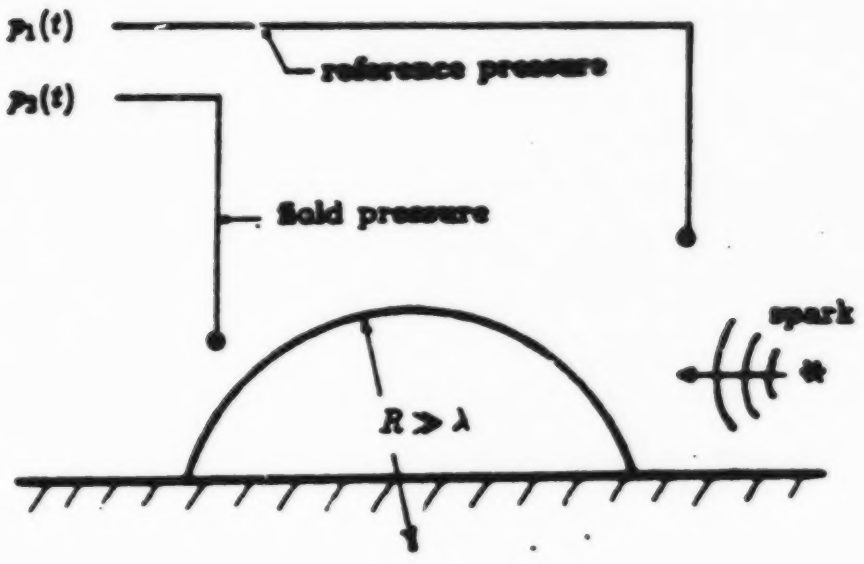


Simplified Diffraction Model

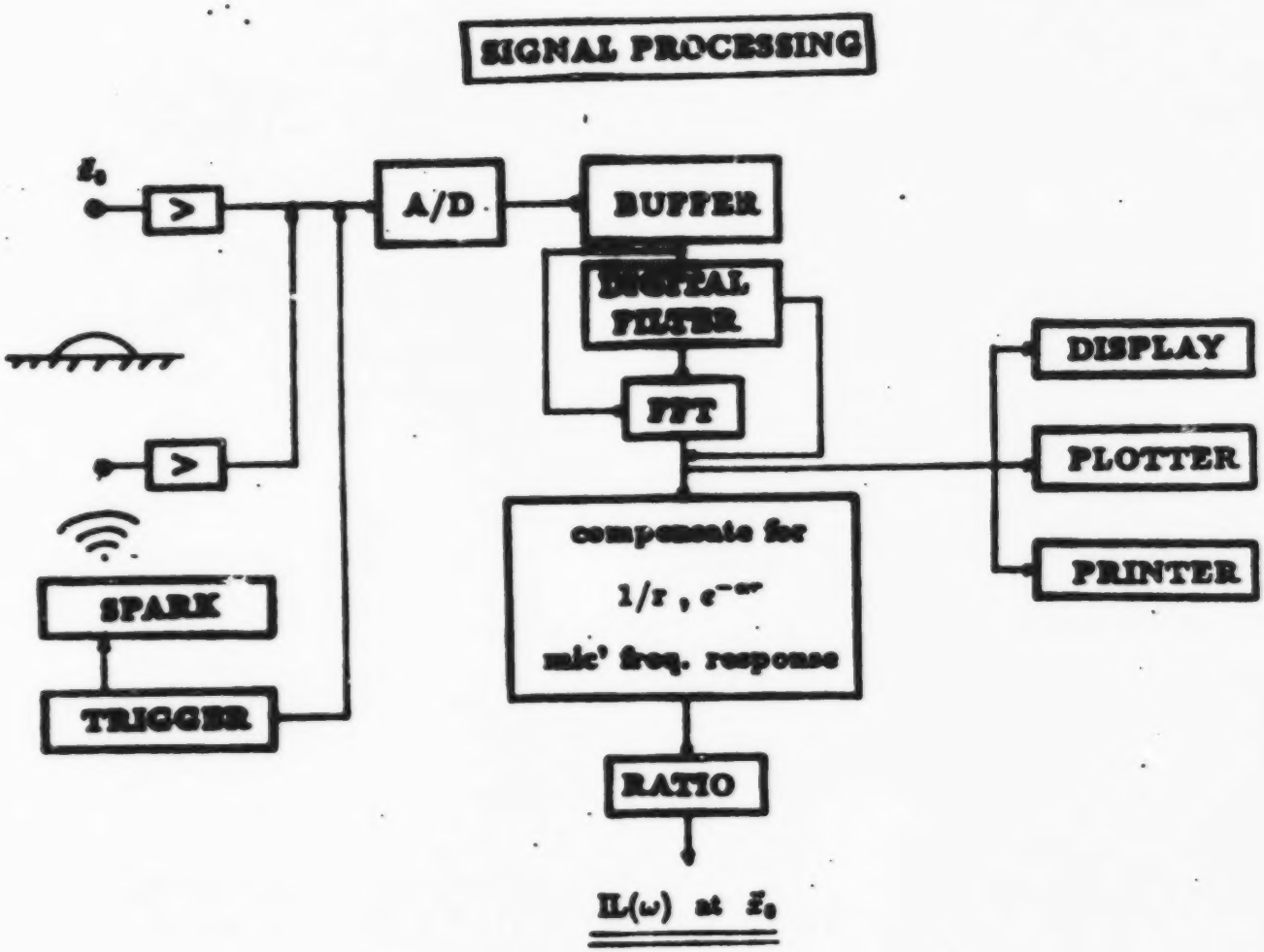
- Irregular topography (hills, valleys)
- Stratified atmosphere

EXPERIMENTAL METHOD



THE RATIO $p_2(\omega)/p_1(\omega)$ IS INDEPENDENT OF THE SPARK SIGNATURE.

IT IS THEREFORE A MEASURE OF THE INSERTION LOSS OF THE RIDGE.


SIGNAL PROCESSING
$\vec{x}_0$
A/D
BUFFER
DIGITAL FILTER
FFT
DISPLAY
PLOTTER
PRINTER
compensate for
1/r , $e^{-\alpha r}$
mic' freq. response
RATIO
SPARK
TRIGGER
$H_L(\omega)$ at $\vec{x}_0$

INNER SOLUTION: wave diffraction

$$\begin{cases} \nabla^2 p + k^2 p = 0 \\ \dfrac{\partial p}{\partial n} + Qp = 0 \end{cases}$$ on the surface (parabola)

with $Q = ik\rho c/Z$

Change to parabolic cylinder coordinates ξ, η

$$\begin{cases} i\dfrac{\partial G}{\partial \xi} + \dfrac{\partial^2 G}{\partial \eta^2} + \eta G = 0 \\ \dfrac{\partial G}{\partial \eta} + qG = 0 \end{cases}$$ at $\eta = 0$

with:

$$p = e^{ikx} e^{ik^2/2} G(\xi, \eta, q)$$

$$q = i\left(\frac{kR}{2}\right)^{1/2} \frac{\rho c}{Z}$$

GENERAL SOLUTION

MATCHING PRINCIPLE: AT LARGE η THE INNER SOLUTION MUST MATCH THE OUTER SOLUTION.

$$p = P_1\, e^{\frac{1}{4}i\pi}\, e^{ikr}/r\; G(\xi, \eta, q)$$

$$G(\xi, \eta, q) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \left[v(\alpha - \eta) - \frac{v'(\alpha) - qv(\alpha)}{w_1'(\alpha) - qw_1(\alpha)} w_1(\alpha - \eta) \right] e^{i\alpha\xi} d\alpha$$

$$v(x) = \pi^{1/2} \mathrm{Ai}(x)$$

$$w_1(x) = e^{i\pi/3}\, 2\pi^{1/2}\, \mathrm{Ai}(x e^{i\pi/3})$$

FOCK - VAN DER POL - BREMMER DIFFRACTION INTEGRAL

LIMITING CASES

1) Shadow zone $(\xi > q^{1/3})$

$$G(\xi,\eta,q) = 2i\pi \sum(\text{residues at poles of integrand})$$
$$= \sum(\text{creeping waves})$$

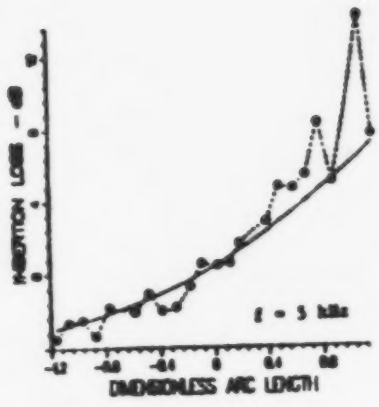
It reduces to the results of Keller (1956) and Hopkit *et al.* (1978).

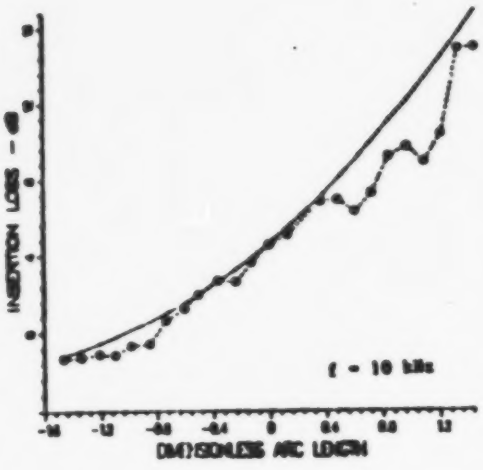
2) The sound pressure on the surface of a ridge

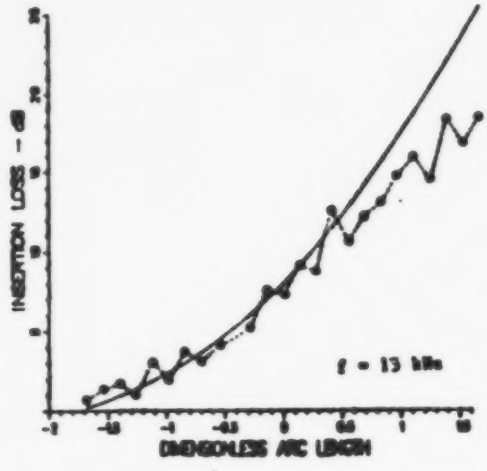
$$G(\xi,0,q) = \pi^{-1/2} \int_0^\infty \frac{e^{-i\beta\ell/2} e^{-\beta\ell 3^{1/3}/2}}{w_2'(\beta) - e^{i2\pi/3} q w_2(\beta)}\, d\beta$$
$$+ \pi^{-1/2} \int_0^\infty \frac{e^{i\alpha\ell}}{w_1'(\alpha) - q w_1(\alpha)}\, d\alpha$$

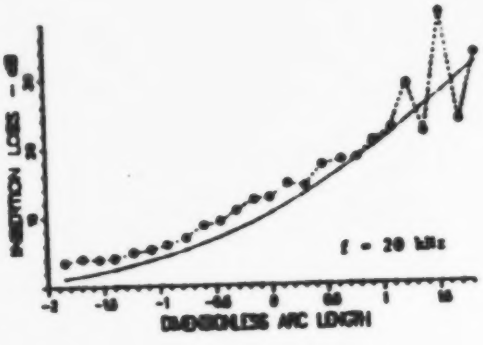
$$w_1(z) = e^{i\pi/6} 2\pi^{1/2} \mathrm{Ai}(z e^{i2\pi/3}) \qquad w_1'(z) = e^{i5\pi/6} 2\pi^{1/2} \mathrm{Ai}'(z e^{i2\pi/3})$$
$$w_2(z) = e^{-i\pi/6} 2\pi^{1/2} \mathrm{Ai}(z e^{-i2\pi/3}) \qquad w_2'(z) = e^{-i5\pi/6} 2\pi^{1/2} \mathrm{Ai}'(z e^{-i2\pi/3})$$

INSERTION LOSS ON THE DIFFRACTING SURFACE









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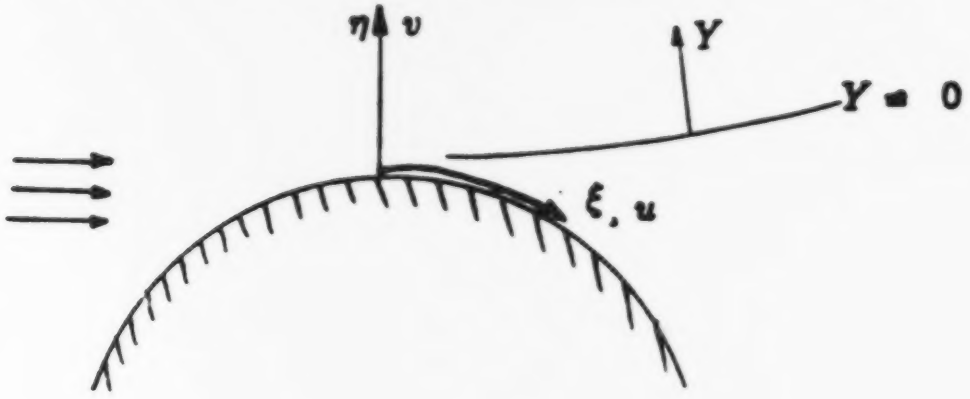
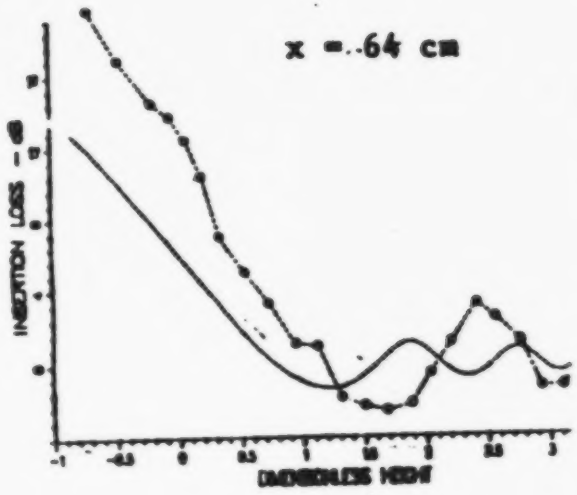
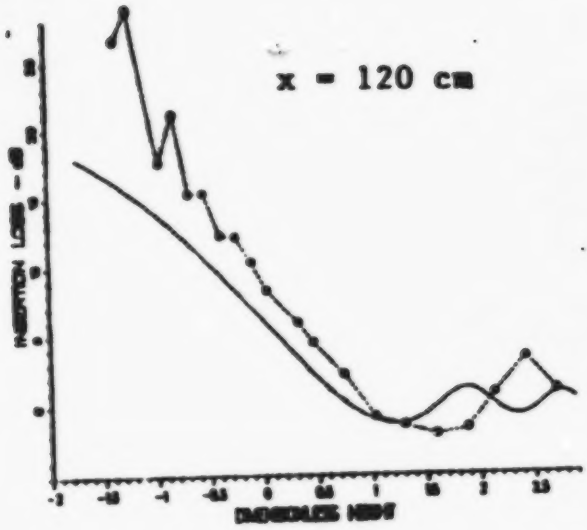
3) The sound pressure in the transition region

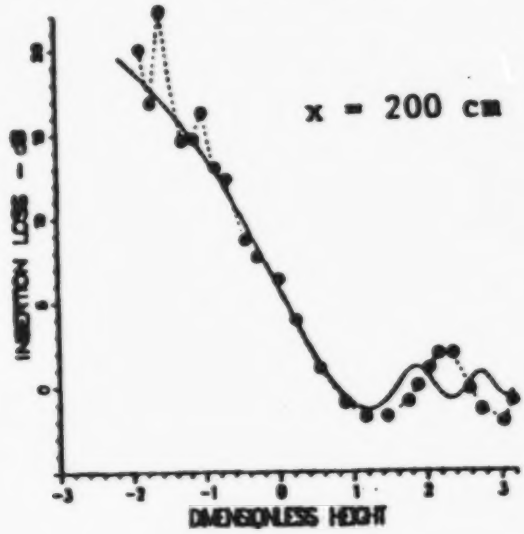
$$G = e^{-i\xi^3/3}e^{i\xi\eta} - e^{i(2/3)\eta^{3/2}}\left[H(Y)e^{-i(\pi/2)Y^2} - \frac{1+i}{2}A_D(Y)\right]$$
$$- \frac{1}{\sqrt{\pi}}\int_0^\infty e^{is\xi e^{i2\pi/3}}\frac{v'(s) - qe^{i2\pi/3}v(s)}{w_2'(s) - qe^{i2\pi/3}w_2(s)}\,w_2(s - \eta e^{-i2\pi/3})ds$$
$$- \frac{1}{\sqrt{\pi}}\int_0^\infty e^{i\alpha\xi}\frac{v'(\alpha) - qv(\alpha)}{w_1'(\alpha) - qw_1(\alpha)}\,w_1(\alpha - \eta)d\alpha,$$

$Y = -(2/\pi)^{1/2}\eta^{1/4}(\xi - \eta^{1/2})$ is a dimensionless height




x = .64 cm
INSERTION LOSS - dB
DIMENSIONLESS HEIGHT


x = 120 cm
INSERTION LOSS - dB
DIMENSIONLESS HEIGHT


x = 200 cm
INSERTION LOSS - dB
DIMENSIONLESS HEIGHT

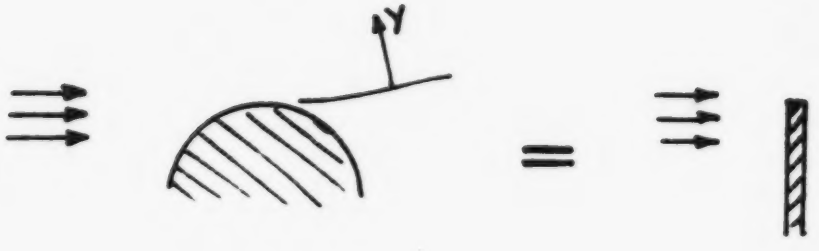
Slide 17

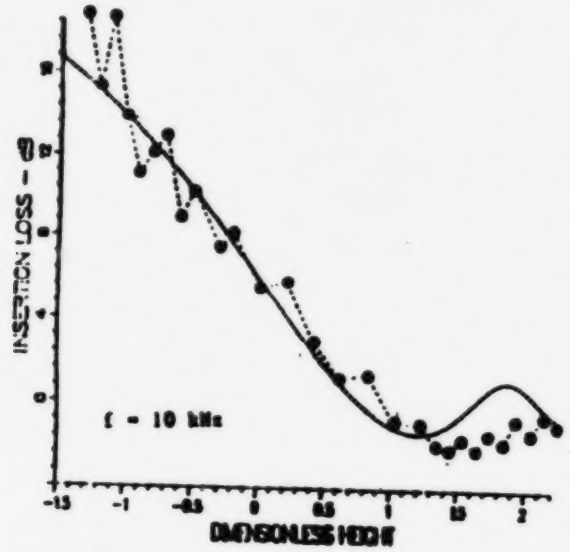
4) Far behind the ridge in the penumbra

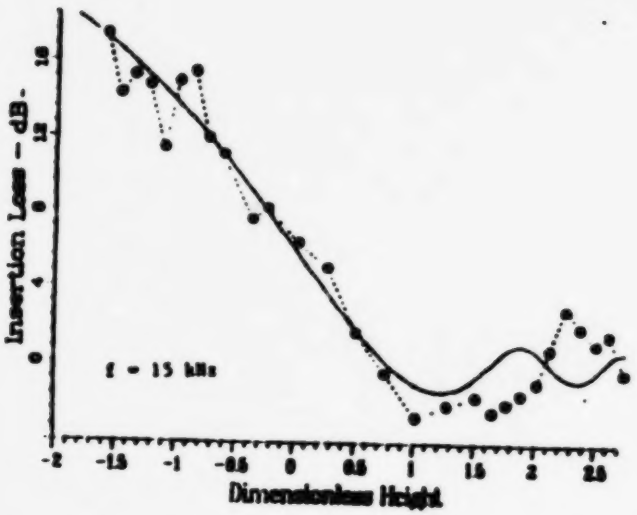
$$G = e^{-i\xi^3/3} e^{i\xi\eta} \left[H(Y) e^{-i(\pi/2)X^3} - \frac{1+i}{2} A_D(Y) \right]$$

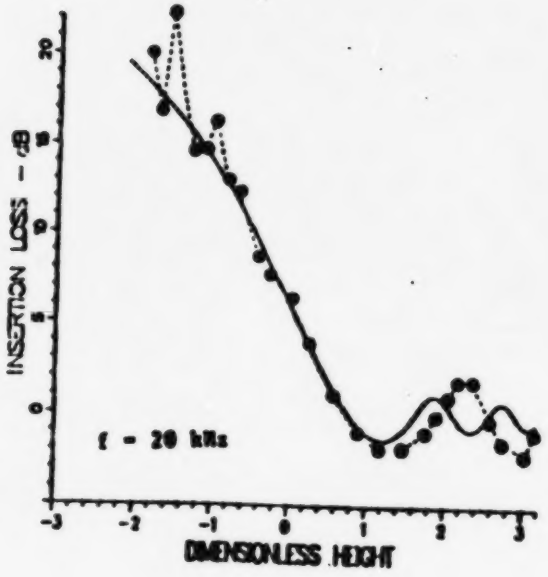
where $Y = (2/\pi)^{1/2} \eta^{1/4} (\xi - \eta^{1/2})$

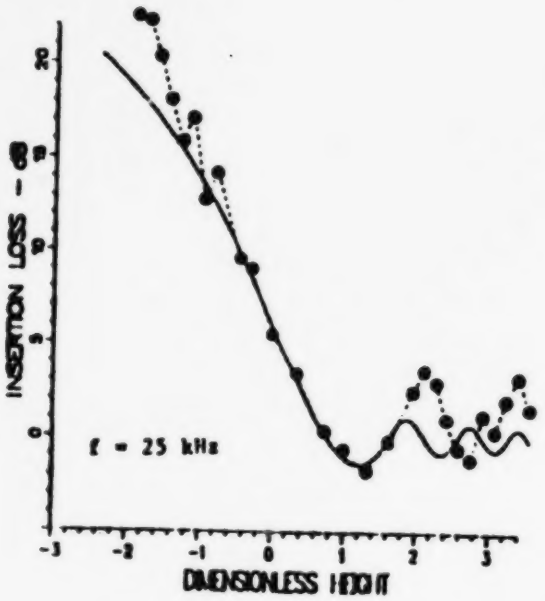
- Knife-edge diffraction
- Independent of the shape and impedance of the hill

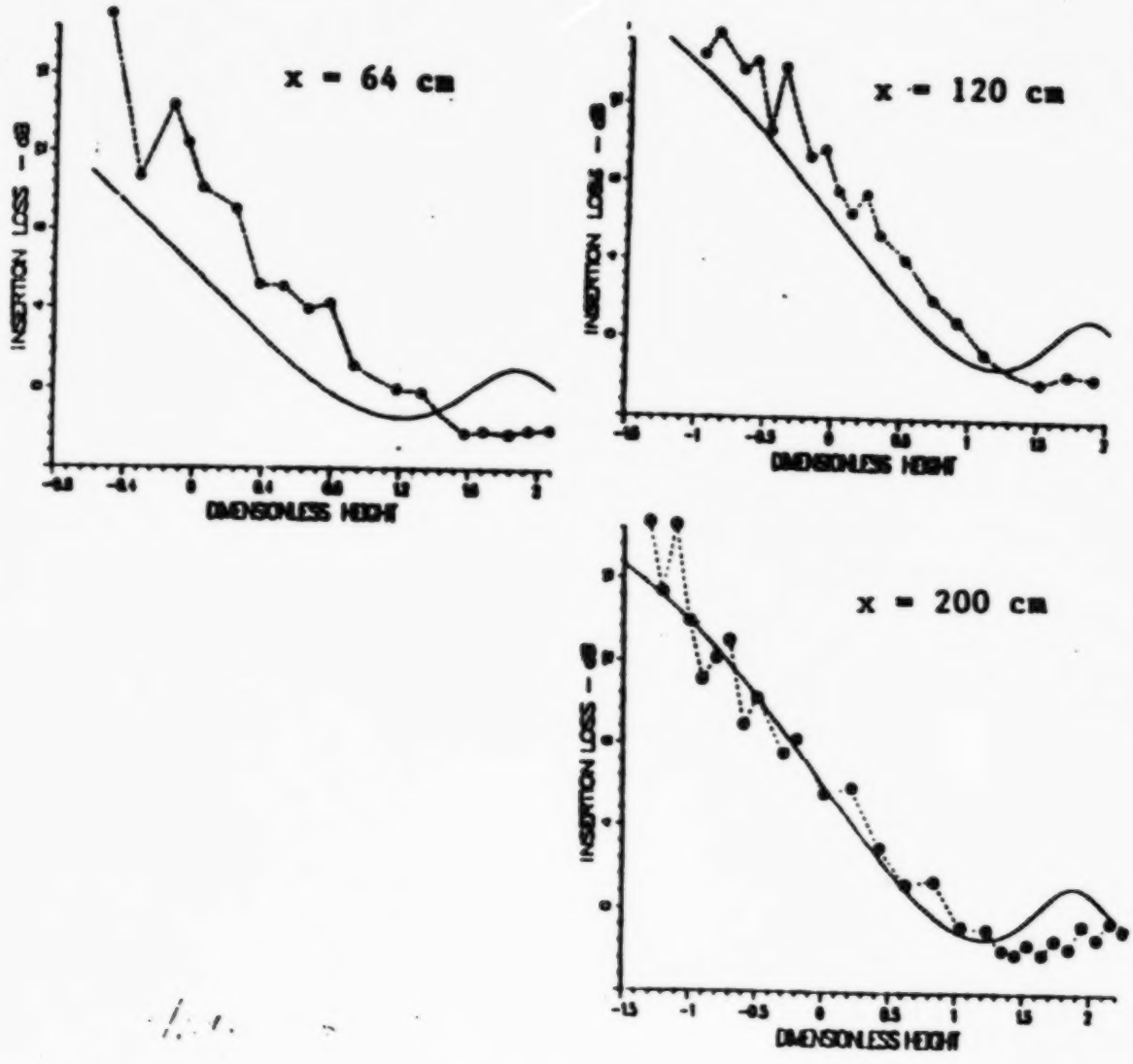



INSERTION LOSS - dB
f = 10 kHz
DIMENSIONLESS HEIGHT


Insertion Loss - dB
f = 15 kHz
Dimensionless Height


INSERTION LOSS - dB
f = 20 kHz
DIMENSIONLESS HEIGHT

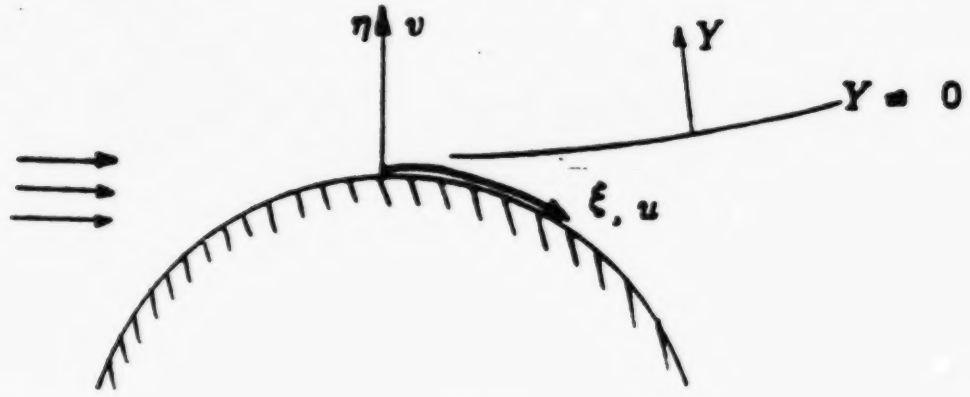

INSERTION LOSS - dB
f = 25 kHz
DIMENSIONLESS HEIGHT

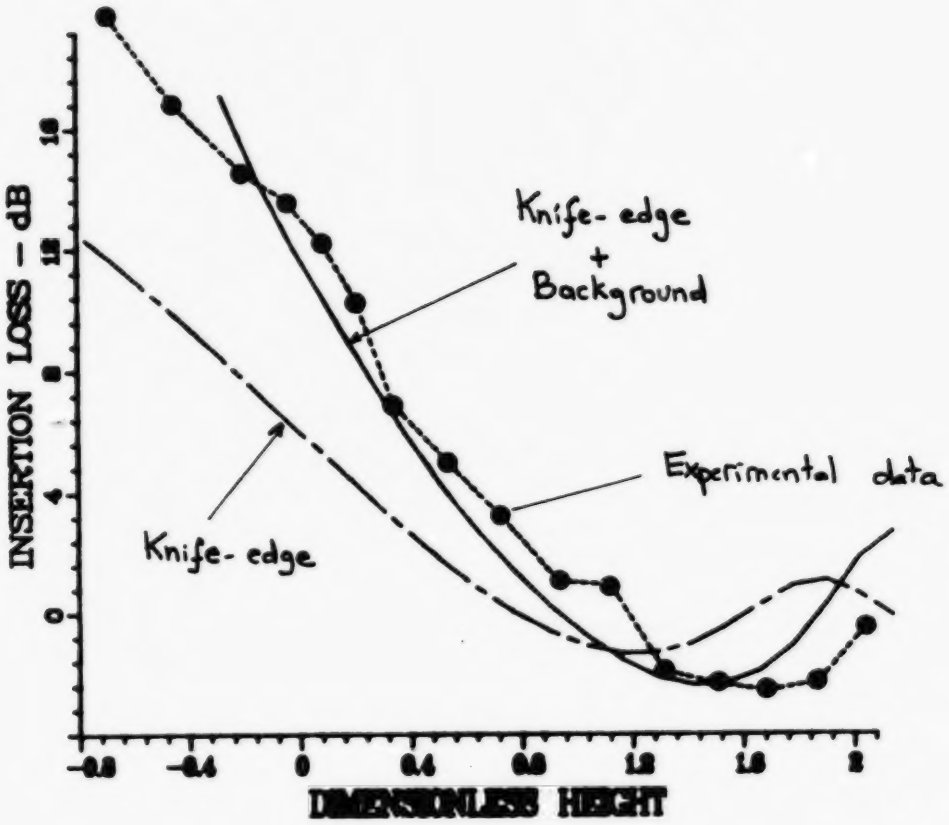

x = 64 cm
INSERTION LOSS - dB
DIMENSIONLESS HEIGHT
x = 120 cm
INSERTION LOSS - dB
DIMENSIONLESS HEIGHT
x = 200 cm
INSERTION LOSS - dB
DIMENSIONLESS HEIGHT

3) The sound pressure in the transition region

$$G = e^{-i\xi^3/3}e^{i\xi\eta} - e^{i(2/3)\eta^{3/2}}\left[H(Y)e^{-i(\pi/2)Y^3} - \frac{1+i}{2}A_D(Y)\right]$$
$$- \frac{1}{\sqrt{\pi}}\int_0^\infty e^{is\xi e^{i2\pi/3}}\frac{v'(s) - qe^{i2\pi/3}v(s)}{w_2'(s) - qe^{2\pi/3}w_2(s)}\,w_2(s - \eta e^{-i2\pi/3})ds$$
$$- \frac{1}{\sqrt{\pi}}\int_0^\infty e^{i\alpha\xi}\frac{v'(\alpha) - qv(\alpha)}{w_1'(\alpha) - qw_1(\alpha)}\,w_1(\alpha - \eta)d\alpha,$$

$Y = -(2/\pi)^{1/2}\eta^{1/4}(\xi - \eta^{1/2})$ is a dimensionless height




INSERTION LOSS – dB
DIMENSIONLESS HEIGHT
Knife-edge
+
Background
Experimental data
Knife-edge

Rigid surface $\sigma = 80,000$ cgs-Rayls

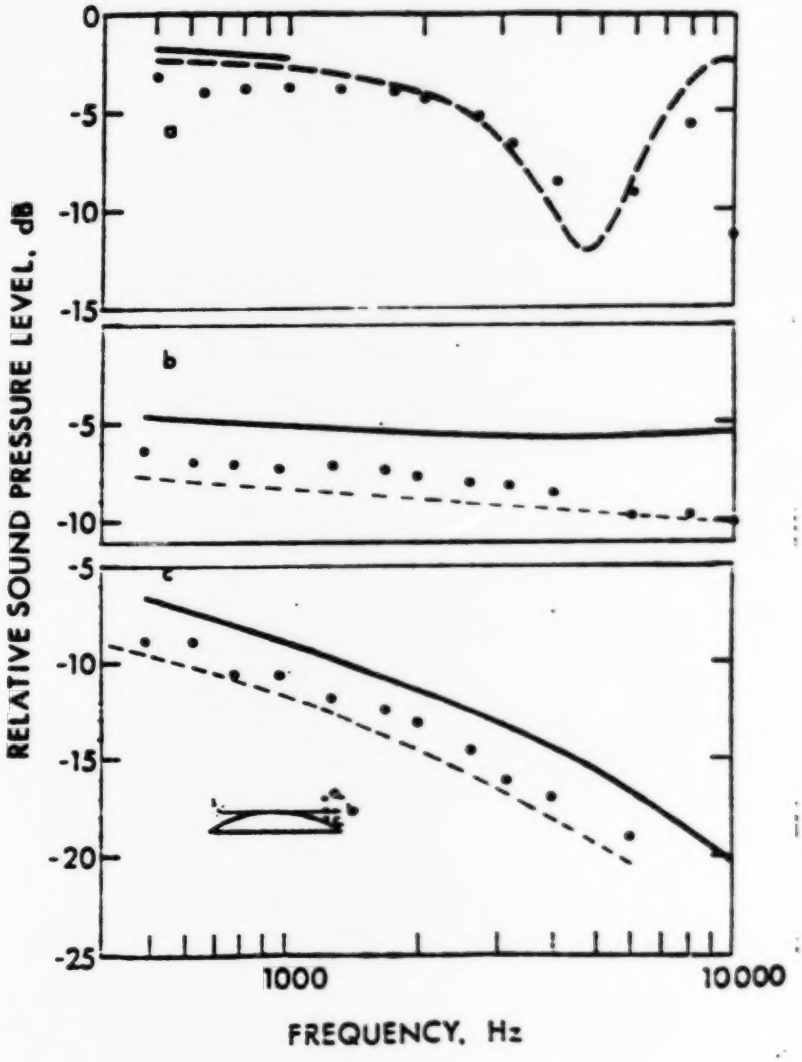


Figure 3 : Relative Sound Pressure Level in the penumbra

• • • : **Data from Berry and Daigle [11]**

——— : **Prediction from the creeping wave series [11]**

- - - - - : **Prediction from MAE theory [Eq. (3)]**

Felt $\sigma = 60$ cgs-Rayls

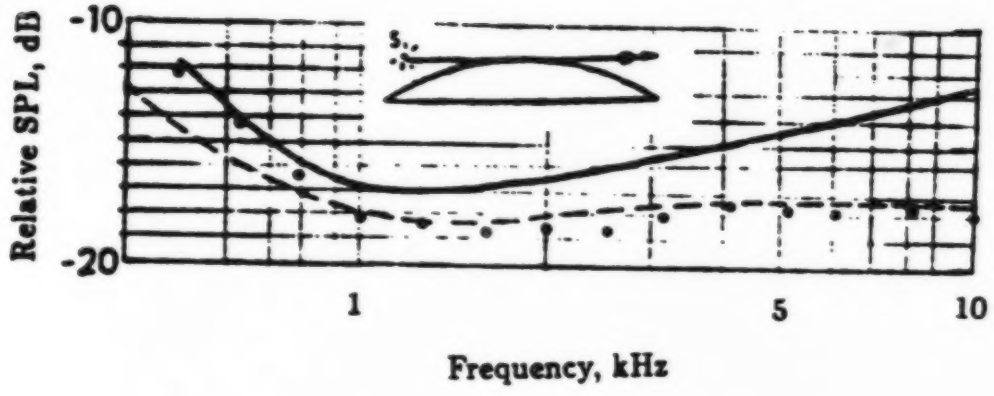


Figure 3 : Relative Sound Pressure Level in the penumbra

• • • : Data from Berry and Daigle [11]

——— : Prediction from the creeping wave series [11]

- - - - - : Prediction from MAE theory [Eq. (3)]

CONCLUSIONS

- MAE Theory [Eq.(3)] is accurate to within 1 dB in the penumbra region, even close to the apex.

- Far behind the ridge, in the penumbra region, knife-edge diffraction formula is accurate to within 0.5 dB.

- In the penumbra, MAE theory is more appropriate than creeping wave series; predictions are within 1 dB of Berry and Daigle's data.

PAPER PRESENTED AT THE 115th ASA MEETING

The following pages reprint a paper that has been presented at the 114th meeting of the Acoustical Society of America, held in Seattle, WA, May 16-20, 1988. James A. Kearns gave the paper. The proper citation for this paper is as follows.

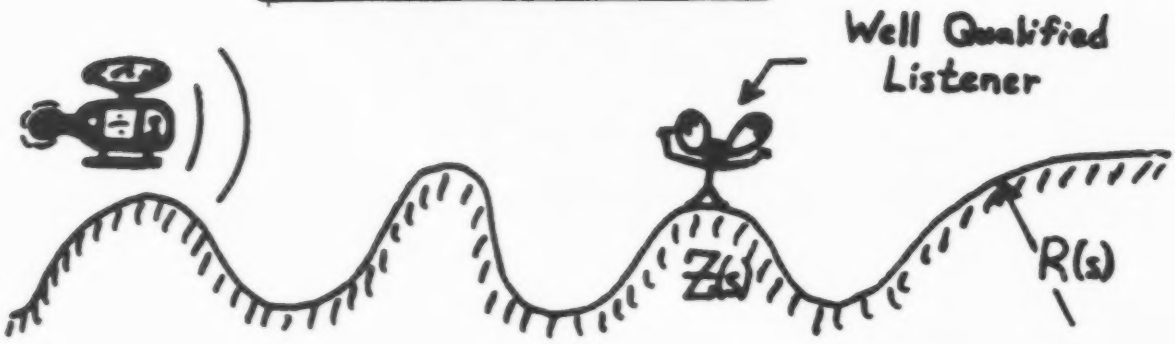
James A. Kearns, Ji-xun Zhou, Yves H. Berthelot, and Allan D. Pierce, "Computational studies of the diffraction integral occuring in the MAE theory of sound propagation over hills and valleys," presented at the the 115 th meeting of the Acoustical Society of America, Seattle, WA (1988). Abstract in *J. Acoust. Soc. Am.* Suppl. 1, **83**, S93 (1988).

COMPUTATIONAL STUDIES OF THE DIFFRACTION INTEGRAL OCCURING IN THE MAE THEORY OF SOUND PROPAGATION OVER HILLS AND VALLEYS

James A. Kearns, Ji-xun Zhou
Yves H. Berthelot, and Allan D. Pierce

School of Mechanical Engineering
Georgia Institute of Technology
Atlanta, Georgia, 30332-0405

Motivation



Sound Propagation over Undulating Terrain

Key Features

1. Distant Source $\longrightarrow$ Grazing Angles of Incidence

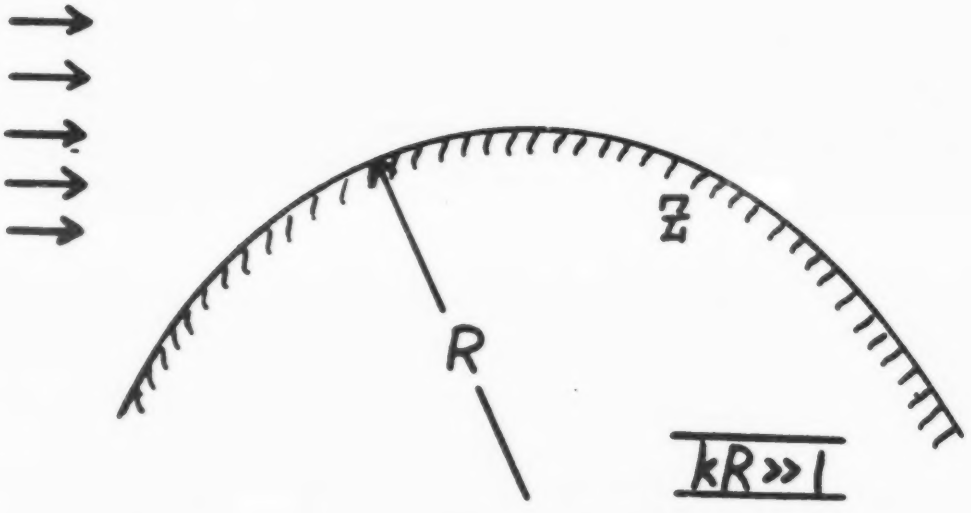
2. High Frequency Propagation $\longrightarrow$ $\lambda \ll R(s)$

3. Surface of Finite Impedance $\longrightarrow$ $Z(s) = |Z(s)|e^{i\theta}$

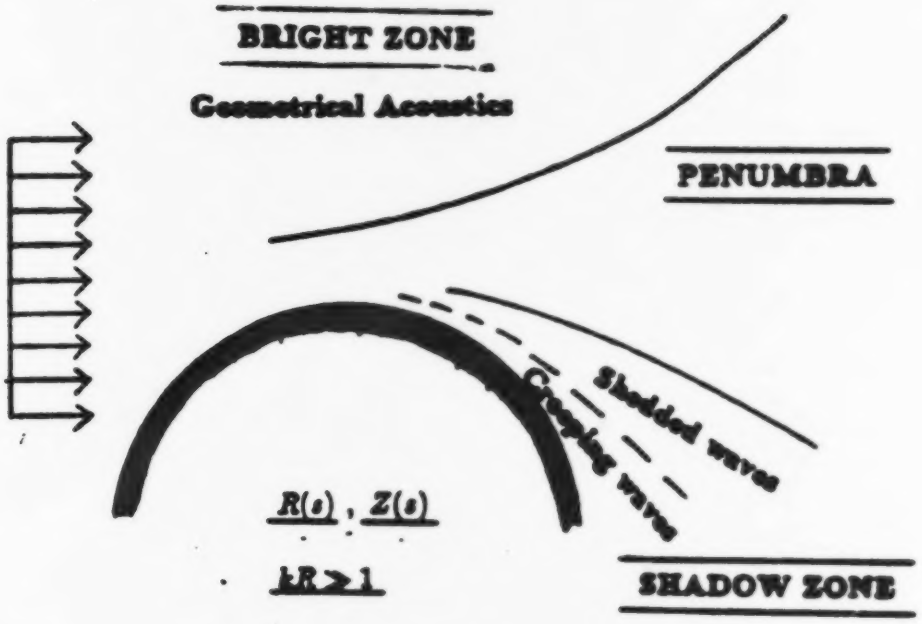
$|Z(s)| < \infty$

Prototype Problem

Plane Wave incident upon a curved surface of slowly varying curvature.



Conceptual Regimes near Curved Surface



Bright Zone

Geometrical (Ray) Acoustics Solution
→ "Outer Solution"

Other Regions

Solution based upon the Helmholtz Equation
and an Impedance Boundary Condition
→ "Inner Solution"

Inner Solution

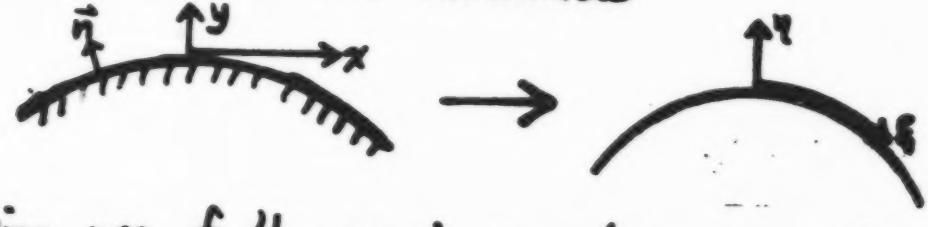
Begin with

$$\nabla^2 P + k^2 P = 0$$

and $\vec{\nabla} P \cdot \vec{n} + QP = 0$ on the surface,

where $Q = \dfrac{ik\rho c}{Z_s}$.

Transform to parabolic coordinates



making use of the previous scaling parameters.

After eliminating higher order terms in $(kR)^{-1}$, obtain

$$i\frac{\partial G}{\partial \xi} + \frac{\partial^2 G}{\partial \eta^2} + \eta G = 0$$

and $\dfrac{\partial G}{\partial \eta} + q G = 0$ at $\eta = 0$, $q = i\left(\dfrac{kR}{2}\right)^{1/3}\dfrac{\rho c}{Z_s}$

where $p = P_I e^{ikw} e^{i\xi/3} G(\xi, \eta, q)$.

Solution:

$$G = \int_C e^{i\alpha\xi} g(\alpha)\left[v(\alpha-\eta) - \frac{v'(\alpha) - q\,v(\alpha)}{w_1'(\alpha) - q\,w_1(\alpha)} w_1(\alpha-\eta)\right] d\alpha$$

Fock Function:

$v''(\alpha) = \alpha\, v(\alpha)$

$w_1''(\alpha) = \alpha\, w_1(\alpha)$

Complete Solution

Method of Matched Asymptotic Expansions
of Inner and Outer solutions

Require

$G(\xi,\eta,q) \longrightarrow$ Geometrical Acoustics Solution

as $\eta \rightarrow$ large , and $\xi \rightarrow$ negative.

Result:

$$G(\xi,\eta,q) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{i\alpha\xi} \left[w(\alpha-\eta) - \frac{w'(\alpha) - q\,w(\alpha)}{w_1'(\alpha) - q\,w_1(\alpha)} w_1(\alpha-\eta) \right] d\alpha$$

"Fock - van der Pol - Bremmer Function"

Propose to find solutions of this function in the various regimes around the curved surface.

Limiting Cases

1) **Shadow Zone** ($\xi > \eta^{1/2}$)

Use Cauchy's Residue Theorem

$$G(\xi,\eta,g) = 2\pi i \sum (\text{residues at poles of integrand})$$

$$= \sum (\text{creeping waves})$$

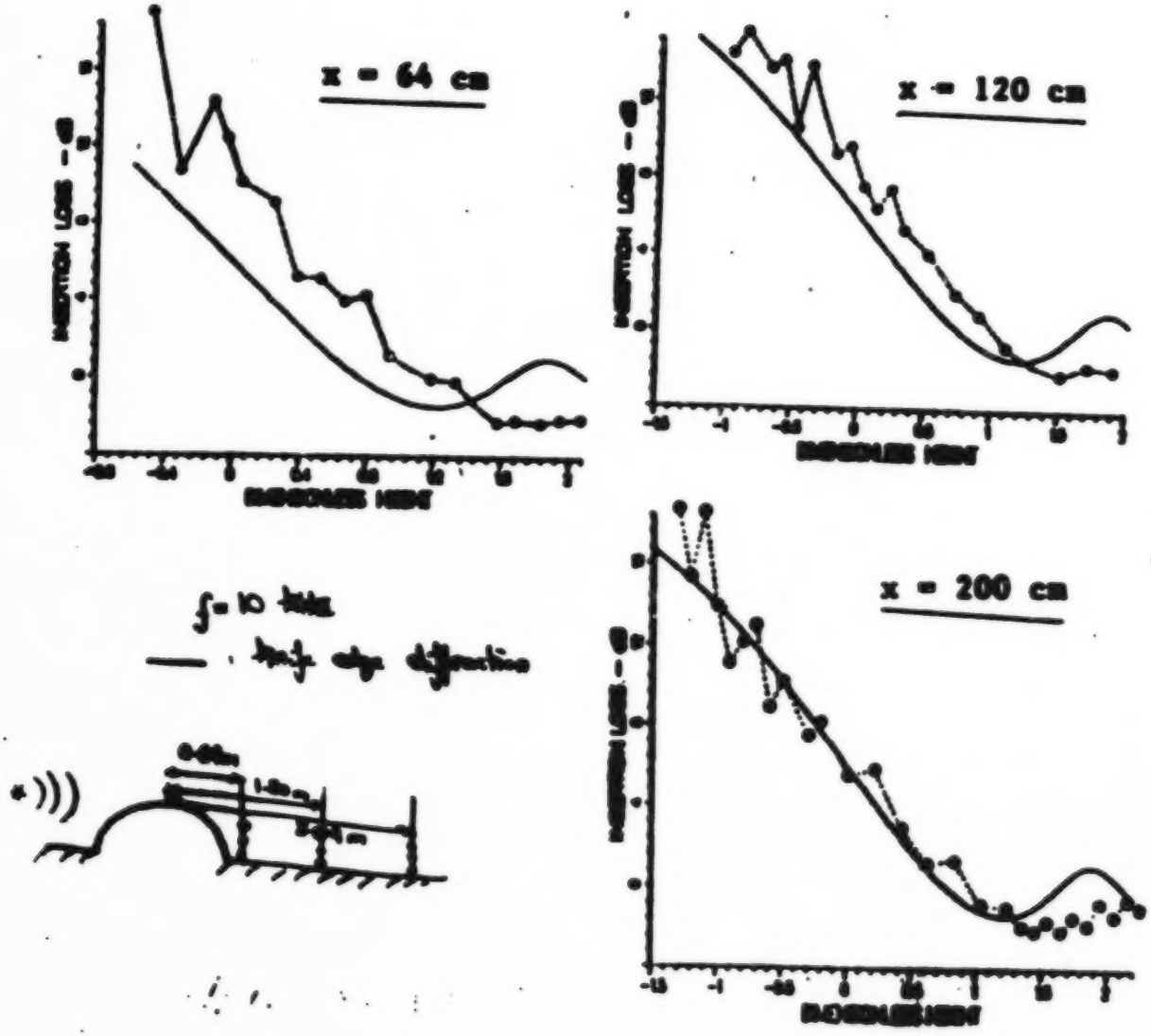
2) **Penumbra** ($\xi \approx \eta^{1/2}$)($\eta > 1$)

$$G \approx e^{-i\xi^3/3} e^{i\xi\eta} - e^{i\frac{2}{3}\eta^{3/2}}\left[H(X)e^{-i\frac{\pi}{2}X^2} + \frac{1+i}{2}A_0(X)\right]$$

where $X = -\left(\frac{2}{\pi}\right)^{1/2}\eta^{1/4}(\xi - \eta^{1/2})$.

$A_0(X)$ is the diffraction integral associated with the asymptotic solution for diffraction from a wedge when (kr and kr_2) $\gg 1$.

→ "Knife Edge Diffraction"



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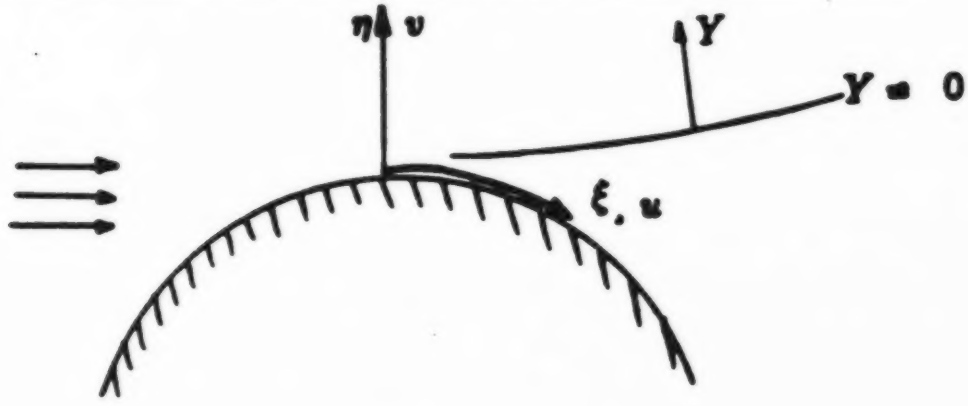
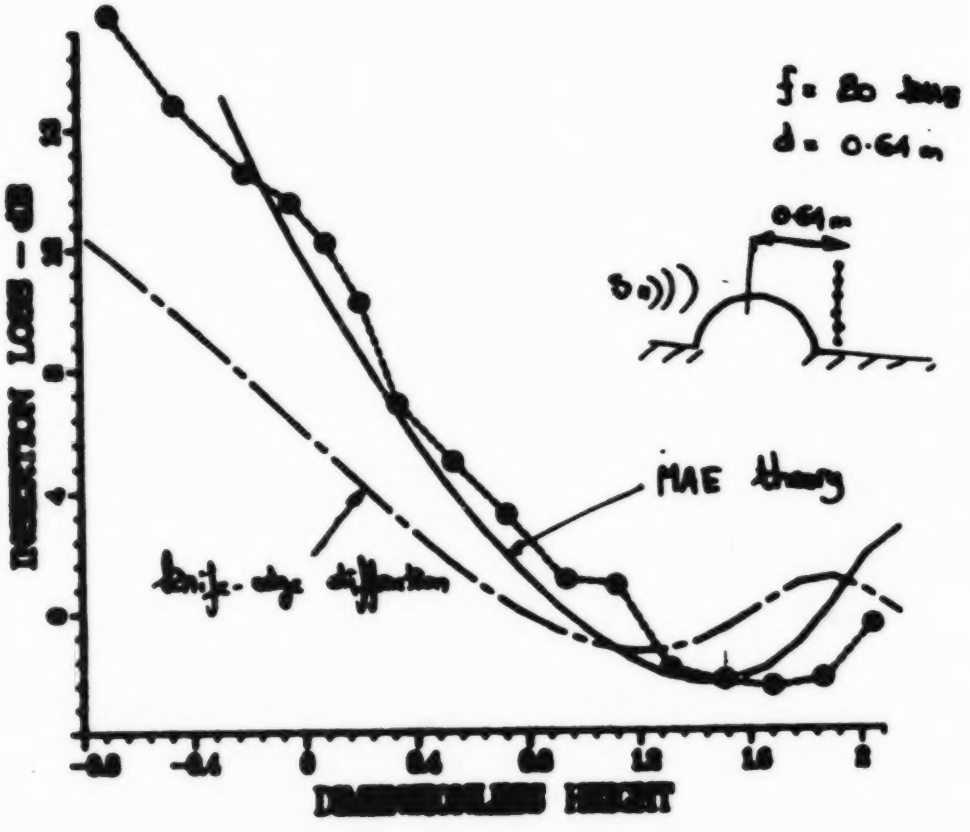
3) The sound pressure in the transition region

$$G = e^{-i\ell^3/3} e^{i\ell\eta} - e^{i(2/3)\eta^{3/2}} \left[H(Y) e^{-i(\pi/3)Y^3} - \frac{1+i}{2} A_D(Y) \right]$$
$$- \frac{1}{\sqrt{\pi}} \int_0^\infty e^{i s \ell e^{i2\pi/3}} \frac{v'(s) - q e^{i2\pi/3} v(s)}{w_2'(s) - q e^{i2\pi/3} w_2(s)} w_2(s - \eta e^{-i2\pi/3}) ds$$
$$- \frac{1}{\sqrt{\pi}} \int_0^\infty e^{i\alpha\ell} \frac{v'(\alpha) - q v(\alpha)}{w_1'(\alpha) - q w_1(\alpha)} w_1(\alpha - \eta) d\alpha,$$

$Y = -(2/\pi)^{1/2} \eta^{1/4} (\xi - \eta^{1/2})$ is a dimensionless height




f = 80 kHz
d = 0.64 m
0.64 m
MAE theory
INSERTION LOSS - dB

Felt $\sigma = 60$ cgs-Rayls

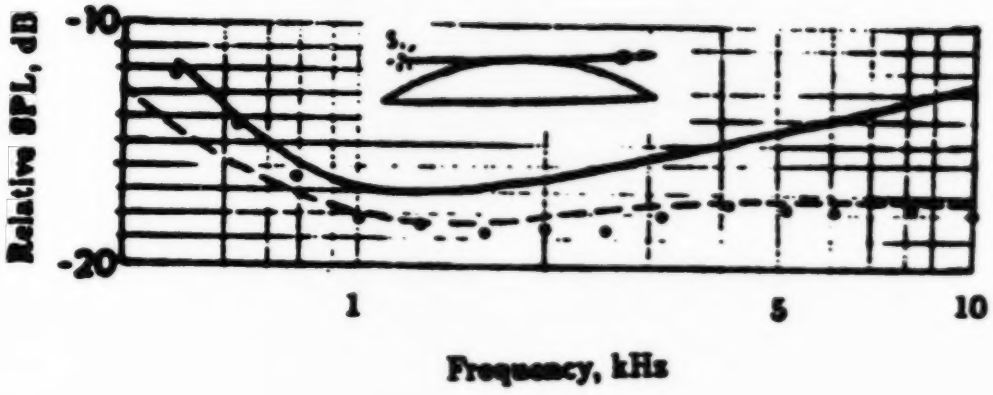
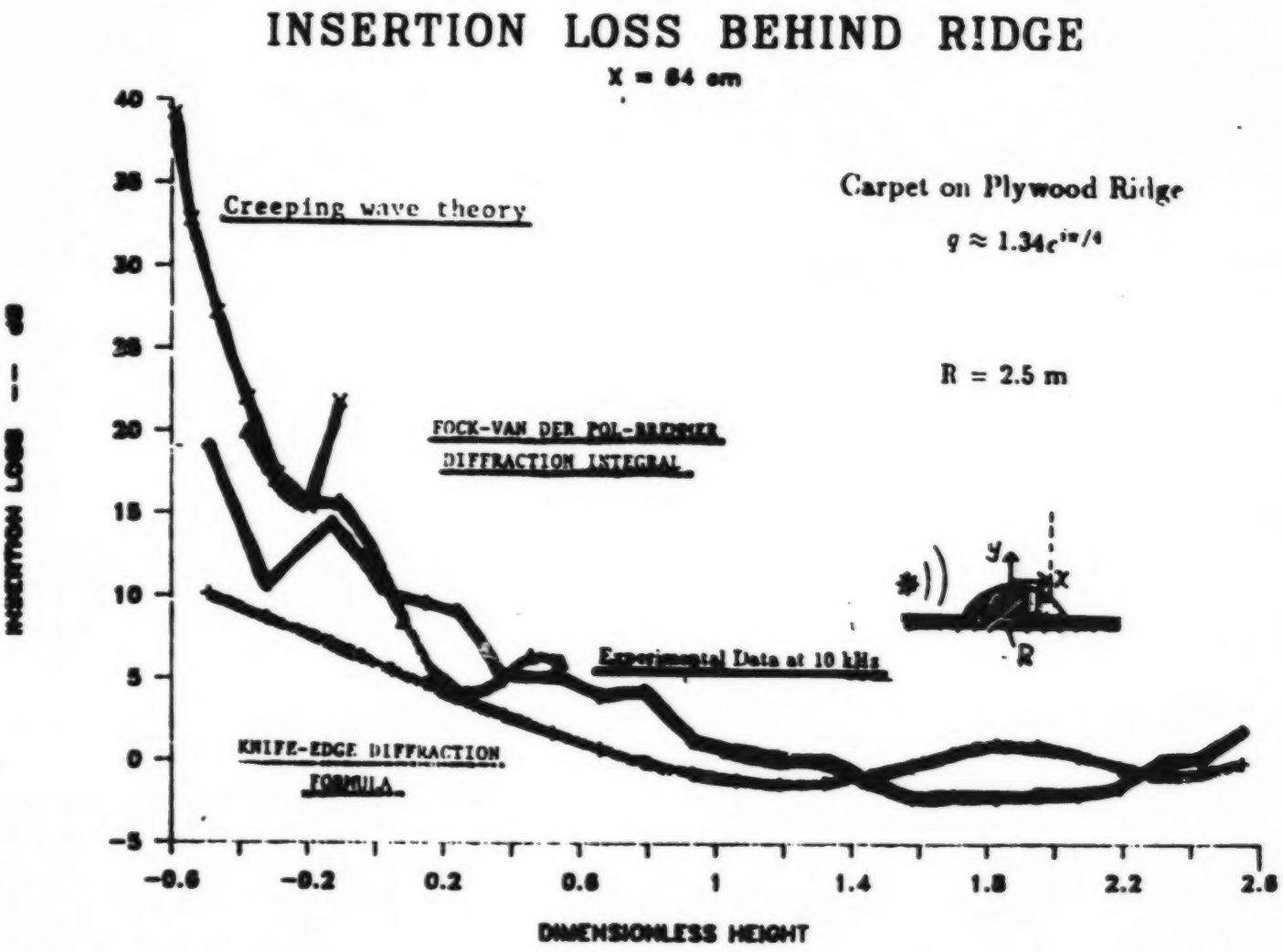



Figure 3 : Relative Sound Pressure Level in the penumbra

• • • : Data from Berry and Daigle [11]

——— : Prediction from the creeping wave series [11]

- - - - - : Prediction from MAE theory [Eq. (3)]


INSERTION LOSS BEHIND RIDGE
X = 84 cm
Carpet on Plywood Ridge
$q \approx 1.34e^{i\pi/4}$
R = 2.5 m
Creeping wave theory
FOCK-VAN DER POL-BREMMER
DIFFRACTION INTEGRAL
Experimental Data at 10 kHz
KNIFE-EDGE DIFFRACTION
FORMULA
INSERTION LOSS -- dB
DIMENSIONLESS HEIGHT
40
35
30
25
20
15
10
5
0
-5
-0.6
-0.2
0.2
0.6
1
1.4
1.8
2.2
2.6
y
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ABSTRACT OF A FORTHCOMING PRESENTATION

The following abstract has been submitted for presentation at the upcoming 116th meeting of the Acoustical Society of America, to be held in Honolulu, HI, November 14-18, 1988.

Experimental validation of the MAE theory of sound propagation over two consecutive ridges of finite impedance. James A. Kearns, Yves H. Berthelot, Ji-xun Zhou (School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, Georgia, 30332)

A difficult problem within the realm of outdoor sound propagation is predicting the acoustic field associated with long range sound propagation over hilly terrain. Such terrain is often of irregular geometry with variable finite impedance. An appealing approach to handling this problem is to consider the local field around a particular ridge with the hope of splicing together a global solution from a set of local solutions. An initial study was made of the diffraction of a plane wave by a single cylindrical ridge of finite impedance. The method of Matched Asymptotic Expansions (MAE), for large values of kR, (where k is the wavenumber and R the radius of curvature of the ridge), was employed to obtain a solution valid on, near, and behind the ridge. Predictions based upon this solution compared very well with the results of scale model experiments (*J. Acoust. Soc. Am.*, Suppl. 1, 83, S93). Recently, scale model experiments have been conducted to study the propagation of sound over two consecutive identical ridges. Preliminary results indicate that the MAE theory is indeed a powerful tool for the analysis of the diffraction of sound over complicated boundaries. [Work supported by NASA Langley Research Center.]

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